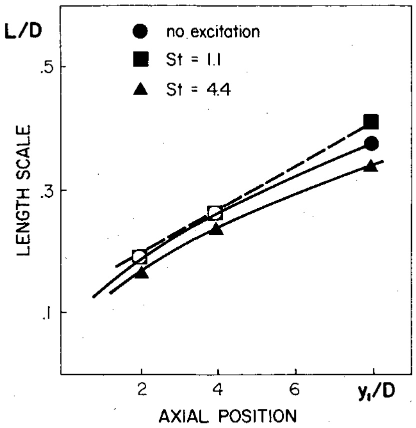

Fig. 2 **Influence of acoustic excitation on axial length scale.**

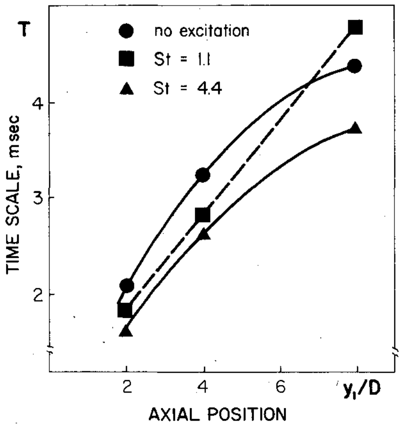

Fig. 3 **Influence of acoustic excitation on the time scale.**

At excitation levels of the order of 120 dB ($\cong 1.6\%$ of 0.5 ρU_j^2) the induced changes in the turbulence levels are approximately 10%. Greater percentage changes are realized near the potential core[11]; this region, however, does not contribute significantly to the radiated sound field.[12]

Figures 2 and 3 summarize the variations of length and time scales as functions of axial position and mode of excitation. The scales increase as the observer moves downstream. This is compatible with the notion that the jet flow diffuses and decelerates.

Excitation at $St = 4.4$ reduces the turbulence levels as well as the length and time scales. Recall that a drop in turbulence level and length scale led to a decrease in the effective source strength; this beneficial effect must compete with the adverse influence of a diminished coherence time; at best, only marginal jet noise suppression can be expected. The present, albeit simplified, model predicts a 0.8 dB enhancement.

For the case of excitation at the moderate Strouhal number of 1.1, an increase in the turbulence level is observed. Length and time scales both exhibit similar behavior, and their influence on the radiated intensity is minimal. This results in an estimated augmentation of 3 dB in the overall sound pressure level. No acoustic measurements were taken to check the prediction, as the apparatus was located in a reverberant environment.

The present results suggest that the potential for turbulence control and the attendant reduction of jet noise is rather poor when a radial (pinching) mode of acoustic excitation is applied in the vicinity of the jet nozzle. Thus one should look to alternate methods of "tickling" the flow, so that the desired changes in the turbulence level, the length, and time scales are evoked.

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Numerical Evaluation of Principal Value Integral by Gauss-Laguerre Quadrature

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Introduction

DURING the past three decades, several numerical methods have been applied to the study of the hydrodynamics of a body floating on a free surface. These numerical methods fall into one of three groups, namely, multipole expansion, finite element (variational principle), and surface source distribution (Green's function).

Among the preceding methods, the surface source distribution method is favorable for a three-dimensional body of arbitrary shape in a uniform depth of water, however, a major difficulty has been encountered in applying this method. This difficulty resides in the evaluation of an improper integral containing a singularity in the Green's function.

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Monacella[1] has proposed a technique by which the integrand of the principal value integral tends to vanish due to the feature of symmetry of its integrand. Faltinsen and Michelsen[2] first rewrote Green's function by adding and subtracting a Green's function corresponding to the source image about the free surface. This results in an expression containing an improper integral whose upper limit is bounded; if the water is sufficiently deep, the resulting expression is integrable. Monacella's procedure is then used to calculate the principal value integral. Hogben and Standing[3] have sought to refine Monacella's method by taking into account the contribution to the principal value integral of the region close to the singularity. Garrison et al.[4-7] used Monacella's singularity removal method, but Monacella's method consumes a large amount of computing time.

Hearn[8] introduced new alternative forms of the principle value integral in the Green's function for infinite depth, but they lack generality of application.

In this Note, the possibility is examined of using Gauss-Laguerre quadrature for the direct calculation of the principal value integral in the Green's function. In this manner, no asymptotic expression of the Green's function for low or high frequencies need be used; consequently, there will be a gain in generality of application, if nothing else.

First, an explanation is given as to how Gauss-Laguerre quadrature can be applied to the numerical evaluation of principal value integral. This is followed by the numerical evaluation of the principal value integral in the Green's function, then a comparison is shown.

Procedure

Gauss-Laguerre quadrature is used to evaluate approximately integrals of the form

$$\int_0^\infty \exp(-x) f(x)\,\mathrm{d}x \simeq \sum_{i=0}^{n} w_i f(x_i) \tag{1}$$

where w_i is a weight factor and x_i the ith zero of the nth Laguerre polynomial $Ln(x)$ (Ref. 9), and where the integrand $f(x)$ is to be bounded. The principal value integral, which contains a singularity, is modified as follows so that the Gauss-Laguerre quadrature technique can be applied.

Consider the principal value integral of the from,

$$PV\int_0^\infty \exp(-x)\frac{f(x)}{g(x)}\,\mathrm{d}x = PV\int_0^\infty \exp(-x)F(x)\,\mathrm{d}x \tag{2}$$

where

$$F(x) = f(x)/g(x)$$

when $g(x)$ has a zero of 1 deg at $x=a$ with the consequence that $F(x)$ is no longer bounded. In the vicinity of the singular point, $F(x)$ is approximately

$$F(x) \simeq F_1(x) = f(a)/[(x-a)g'(a)] \tag{3}$$

Then, Eq. (2) results as follows.

$$PV\int_0^\infty \exp(-x)F(x)\,\mathrm{d}x = \int_0^\infty \exp(-x)\{F(x) - F_1(x)\}\,\mathrm{d}x + PV\int_0^\infty \exp(-x)F_1(x)\,\mathrm{d}x = I_1 + I_2 \tag{4}$$

and Gauss-Laguerre quadrature can be used for obtaining the first integral I_1.

The second integral I_2 has the form of

$$PV\int_0^\infty \frac{\exp(-x)}{x-a}\,\mathrm{d}x = -\exp(-a)E_i(a) \tag{5}$$

where $E_i(x)$ is the exponential integral.[10]

Hence, Eq. (2) becomes

$$PV\int_0^\infty \exp(-x)\frac{f(x)}{g(x)}\,\mathrm{d}x = \int_0^\infty \exp(-x)\left\{\frac{f(x)}{g(x)} - \frac{f(a)}{(x-a)g'(a)}\right\}\mathrm{d}x - \exp(-a)E_i(a)\frac{f(a)}{g'(a)} \tag{6}$$

Following the notation of Monacella,[1] the Green's function is written as

$$Gh(\gamma,\sigma) = \frac{2}{[\gamma^2+1]^{1/2}} + 2PV\int_0^\infty \frac{(\mu+\sigma)e^{-\mu}}{\mu\tanh(\mu)-\sigma}J_0(\mu\gamma)\,\mathrm{d}\mu + 2\pi i\frac{(\mu_0^2-\sigma^2)}{\mu_0^2-\sigma^2+\sigma}J_0(\mu_0\gamma) \tag{7}$$

where μ_0 is the positive, real root of $\mu\tanh(\mu)-\sigma=0$.

Changing the integrand of the principal value integral and defining the principal value integral as follows:

$$H \equiv PV\int_0^\infty \frac{(\mu+\sigma)e^{-\mu}\cosh(\mu)}{\mu\sinh(\mu)-\sigma\cosh(\mu)}J_0(\mu\gamma)\,\mathrm{d}\mu \tag{8}$$

Upon defining

$$F(\mu) \equiv f(\mu)/g(\mu)$$

where

$$f(\mu) = (\mu+\sigma)\cosh(\mu)J_0(\mu\gamma)$$

$$g(\mu) = \mu\sinh(\mu) - \sigma\cosh(\mu)$$

$F(\mu)$ becomes

$$F(\mu) \simeq F_1(\mu) = \frac{(\mu_0+\sigma)\cosh(\mu_0)J_0(\mu_0\gamma)}{(\mu-\mu_0)\{\sinh(\mu_0)+\mu_0\cosh(\mu_0)-\sigma\sinh(\mu_0)\}} \tag{9}$$

near $\mu=\mu_0$.

Finally, H becomes

$$H = \int_0^\infty e^{-\mu}\left[\frac{(\mu+\sigma)\cosh(\mu)J_0(\mu\gamma)}{\mu\sinh(\mu)-\sigma\cosh(\mu)} - \frac{(\mu_0+\sigma)\cosh(\mu_0)J_0(\mu_0\gamma)}{(\mu-\mu_0)\{\sinh(\mu_0)+\mu_0\cosh(\mu_0)-\sigma\sinh(\mu_0)\}}\right]\mathrm{d}\mu - e^{-\mu_0}E_i(\mu_0)\frac{(\mu_0+\sigma)\cosh(\mu_0)J_0(\mu_0\gamma)}{\sinh(\mu_0)+\mu_0\cosh(\mu_0)-\sigma\sinh(\mu_0)} \tag{10}$$

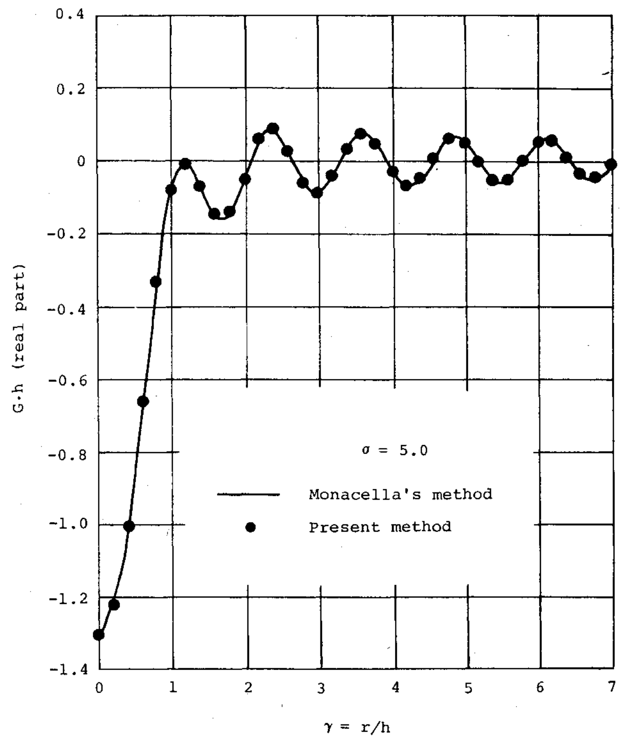

Fig. 1 Comparison between Monacella's method and present method.

Verification and Comparison

To verify this method, the Green's function in Eq. (7) is evaluated for the condition $\mu_0 = 5.000454$ and $\sigma = 5.0$. A comparison is made in Fig. 1 of the real part of the value of Gh obtained by the present technique with that obtained by a technique of Monacella.[1] Gauss-Laguerre quadrature $n = 64$ is used and CPU time is 2.81 s for 36 values in Fig. 1. On the other hand, it takes 32.33 s by Monacella's method. The digital computer used was HITAC M-180. As can be judged, the results are virtually identical and the present method is computationally much faster than Monacella's method.

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Carbon Catalyzed SO_2 Oxidation by NO_2

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Introduction

VARIOUS mechanisms have been suggested to account for the atmospheric transformation of SO_2 to sulfate. These mechanisms have ranged from purely gas phase reactions[1] to reactions in aqueous cloud droplets.[2,3] The catalytic oxidation of SO_2 on particulates has received increased attention since Novakov[4] demonstrated that carbon particles could effectively catalyze SO_2 oxidation to sulfate in air at room temperature. Chang and Novakov[5] subsequently suggested that carbon particles covered with a liquid water phase could dominate the atmospheric conversion of SO_2 to sulfate under certain realistic conditions. Cofer et al.[6,7] then demonstrated that SO_2 oxidation on carbon particles could be enhanced significantly by trace amounts of gaseous NO_2 and O_3. In those experiments, initially dry graphitic carbon particles on the pan of a microbalance were exposed to continuously flowing gaseous mixtures of SO_2/air and SO_2/air/NO_2 (or O_3) at 65% relative humidity and then analyzed for sulfate. A significant enhancement in sulfate production resulted with the incorporation of either NO_2 or O_3 into the SO_2/air mixtures. The initially dry carbon particles, however, were observed to progressively "wet" and agglomerate with increasing exposure time. Thus, it was next to impossible to assess from these experiments the extent that aqueous solution chemistry influenced the SO_2 transformation process on the carbon particles. The assessment was achieved partially by conducting a second set of experiments that contrasted the transformation of SO_2 to sulfate in carbon/water suspensions with pure water blanks. This Note represents a review of the bubbler experiments[8] and a comparison of these results with the results from the microbalance studies.

Experimental

Selected gas mixtures of SO_2 in air or $SO_2/\text{air} + NO_2/N_2$ were bubbled through distilled water and carbon black suspensions in distilled water. Typically, 100 mg of commercial carbon black (BET-N_2 surface area $= 240\ m^2/g$) was suspended in 10 ml of water and placed in one of two matched

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